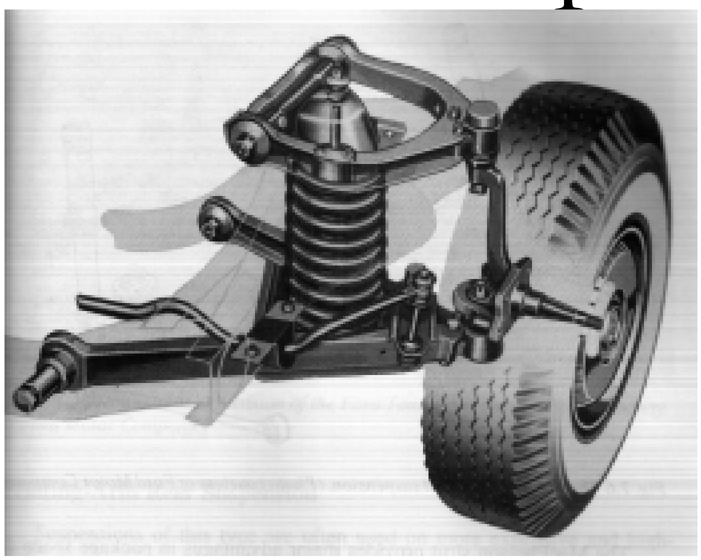
The Helical Spring



Theory and Calculation in the Context of the Automobile Suspension System

CIV 205 Term Project
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Introduction

The suspension of an automobile is a complex system. Not only does it have to support the entire vehicle and its occupants statically, but also it has to withstand dynamic loads that come from accelerating, braking and cornering, road variations and imperfections. Furthermore, everything, from the rigid support control members to the flexible tires and even the seats in an automobile, effects the overall feel that is perceived by the vehicle's occupants.

This CIV 205 project aims to address one of the most important components of the automobile suspension—the helical spring. With some justified simplifications, the analysis of the helical spring is a tractable task. Although the helical (coil) spring is not the only kind of spring found in automobile suspension systems, it is by far the most prevalent among passenger cars, and its role is crucial in maintaining the driveability of cars so equipped with helical springs.

Because of the large number of assumptions required for the analysis of the helical spring, they will not be listed in a specific section. Instead, they will be mentioned together with the analysis, whenever necessary.

After the discussion of background theory is complete, two examples will be given. The first will show how one can evaluate a car with one axle resting on a bump. The other will take into account impact loading, for example, when a wheel hits a similar bump while the car is in motion. The results from these two cases will be then be compared.

General Theory of the Helical Spring

As the helical spring is essentially a twisted bar, the characteristics of the helical spring can be analyzed by taking a section of the helical spring (shown in Figure 1) and analyzing the section as a straight circular bar in torsion. Since the helical automotive spring is used as a compression and support member, one can assume that the spring is loaded only axially, about its central axis. In a fuller analysis, lateral loading and buckling possibilities should also be considered, but they will not be the covered in this project. It is assumed that the spring compresses only in the direction of its axis, with the spring seats holding the ends in place.

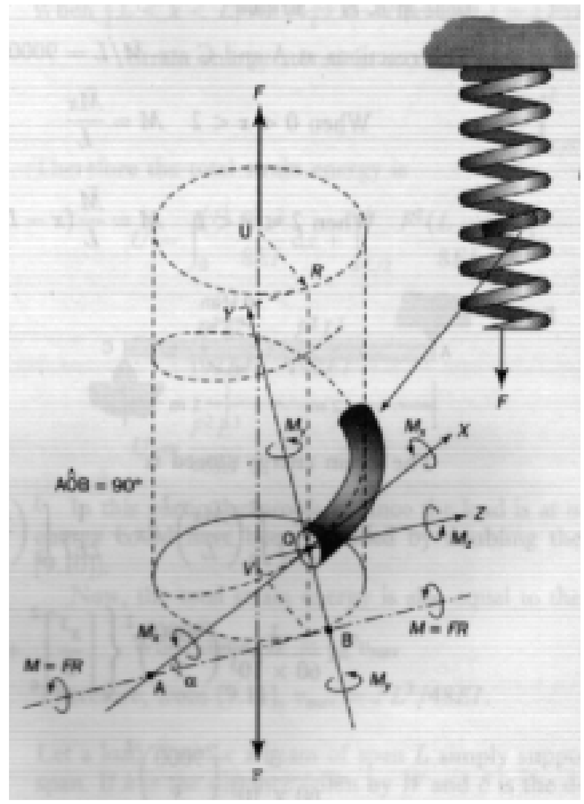

Figure 1. Spring element with forces and moments. (Benham).

With the above assumption of an axially loaded spring, Figure 1 shows the forces and moments on a section of the spring. A simplified view is shown by Figure 2, where the load F is balanced internally by a shearing force V and a torque T to compensate for the fact that F is not applied through the section in consideration. For the purposes of clarity, the uppercase letters R and D will refer to the radius and diameter of the coil, respectively, while the lowercase letters r and d

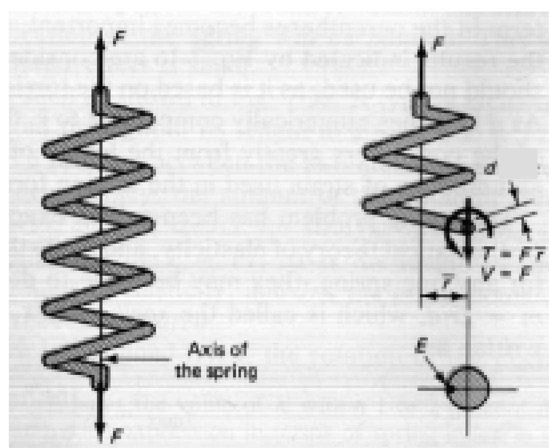

Figure 2. Simplified free body diagram showing a direct shear V and a torque T due to axial loading. (Popov)

will refer to the radius and diameter of the spring element, respectively. Although springs of non-circular cross-sections exist, only the common, circular cross-section springs will be considered.

Superpositioning of Shearing Stresses

$$\tau = \frac{F}{A} + \frac{Tc}{J}$$
$$\tau = \frac{F}{\pi r^2} + \frac{FRr}{\frac{\pi r^2}{2}}$$
$$\tau = \frac{F}{\pi r^2} + \frac{2FR}{\pi r^3}$$
$$\tau = \frac{2FR}{\pi r^3}\left(\frac{r}{2R} + 1\right)$$

Equations 1. Shearing stress as a sum of the direct shear and torsion.

The simplest analysis of the helical spring requires some further assumptions. First, the coils of the spring are assumed to lie in a plane nearly perpendicular to the axis of the spring such that the angle α in Figure 1 is small. This assumption allows the section taken for analysis to be nearly vertical, eliminating the need to consider an axial force and a bending moment at the section taken through the spring. Also, any changes in coil radius are ignored. Thus, there are only two stress contributors; one is the result of the applied force F, and the other is a Torque T that exists because load is collinear with the axis of the coil. By superposition, the two stresses sum and result in the first line of Equations 1. At this point, it should be noted that the first term of Equations 1 (the stress due to the shearing force V) is assumed to be acting uniformly over the cross-section. Note that this averaging results in $\frac{F}{\pi r^2}$ term for the direct shearing stress. In reality, a better approximation would come from using the expression $\frac{VQ}{It}$ and finding Q to the neutral axis and using t as the diameter of the spring element. However, because most texts appear to use the simplified expression $\frac{F}{\pi r^2}$, I will retain this simplification. Since $T = FR$, $c = r$, $A = \pi r^2$ and $J = \pi r^4 / 2$, Equations 1 is simplified, giving the shearing stress on a spring element as the bottom line of Equations 1. As the size of the spring element becomes small with respect to the coil (such that $r << R$), the first term in the parentheses (direct shear) becomes inconsequential. However, when the two radii become comparable, the first term, due to Torque T, cannot be neglected because it is also a significant contributor of the total shearing stress. Nevertheless, a useful equation for the maximum shearing stress does come out of the above analysis if a stress concentration factor, K, is used instead of including the first term of the last line of Equations 1. The result is Equation 2, where the factor K scales the shearing stress appropriately as a function of the ratio of the radii, compensating for the neglect of the direct shearing term. Figure 3 shows how K varies with the spring element and coil size. Wahl proposes that Figure 3 has, as its governing equation, Equation 3. The general formula for the maximum shearing, Equation 2, along with the correction factor K, will be used for future calculations of shearing stress. Note that the correction factor still does not consider the change in radius between the innermost part of the spring element and the outermost part. In reality, stresses will be higher in the inner section of the cross-sectional element, as shown in Figure 4.

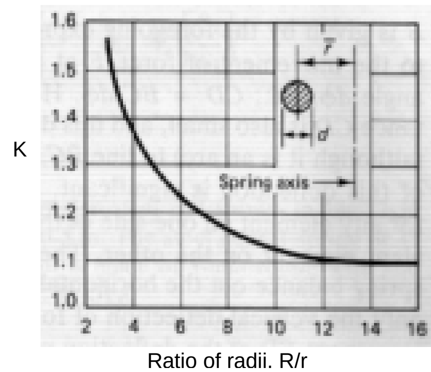

Figure 3. Correction factor K as a function of the ratio of radii, R/r. (Popov)

$$\tau = K\frac{2FR}{\pi r^3}$$

Equation 2. General formula for shear stress using correction factor.

$$K = \frac{4\,R/r - 1}{4\,R/r - 4} + \frac{0.615}{R/r}$$

Equation 3. Correction factor as a function of coil and spring radii.

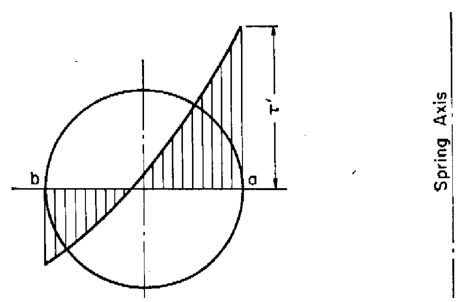

Figure 4. Distribution of shearing stress over a spring element's cross-sectional area. (Wahl)

Energy Method

Although the above analysis based on superpositioning can be used to determine the deflection of a loaded helical spring, and thus its spring rate, an energy balance method is also simple and allows one to see the effects if the coil angle α is significant and cannot be neglected. In this analysis, a standard co-ordinate system (see Figure 1) is used, and a bending moment in addition to the torsion should be taken into account. However, changes in coil radii across the spring element are still considered. From Figure 1, the torsion T or $M_x = FR\cos\alpha$, and the bending moment $M_y = FR\sin\alpha$, while $M_z = 0$. Before continuing, one should note that the work done in deflecting a spring by axial loading, torsion and bending; they are derived in Equations 4a, 4b and 4c, where U is the strain energy and u is the strain energy density (by volume). Equations 4b for torsion is analogous to Equations 4a for axial loading, and Equations 4c for bending can be seen as an extension of axial loading since bending produces a normal stress.

Equations 4a. Energy due to axial loading.

$$U = \int_0^{\delta_1} F d\delta = \int_0^{\delta_1} k\delta d\delta = \tfrac{1}{2}k\delta_1^2$$

$$U = \tfrac{1}{2}F_1\delta_1$$

$$\frac{U}{V} \equiv u$$

$$u = \int_0^{\delta_1} \frac{F}{A}\frac{d\delta}{L} = \int_0^{\varepsilon_1} \sigma d\varepsilon = \int_0^{\varepsilon_1} E\varepsilon d\varepsilon = \frac{E\varepsilon_1^2}{2}$$

$$u = \frac{\sigma_1^2}{2E} = \frac{\sigma^2}{2E}$$

$$\frac{dU}{dV} \equiv u$$

$$dU = udV$$

$$U = \int \frac{\sigma^2}{2E}dV = \int_0^L \frac{F^2}{2A^2E}Ad\delta$$

$$U = \frac{F^2L}{2AE}$$

Equations 4b. Energy due to torsion.

$$U = \int_0^{\theta_1} T d\theta = \int_0^{\theta_1} k\theta d\theta = \tfrac{1}{2}k\theta_1^2$$

$$U = \tfrac{1}{2}T_1\theta_1$$

$$\frac{U}{V} \equiv u$$

$$u = \int_0^{\gamma_1} \tau d\gamma = \int_0^{\gamma_1} G\gamma d\gamma = \frac{G\gamma_1^2}{2}$$

$$u = \frac{\tau_1^2}{2G} = \frac{\tau^2}{2G}$$

$$\frac{dU}{dV} \equiv u$$

$$dU = udV$$

$$U = \int \frac{\tau^2}{2G}dV = \int \frac{T^2\rho^2}{2GJ^2}dV = \int_0^L \frac{T^2}{2GJ^2}\left(\rho^2 dA\right)dx$$

$$U = \frac{T^2L}{2GJ}$$

Equations 4c. Energy due to bending.

$$U = \int \frac{\sigma^2}{2E}dV = \int \frac{M^2y^2}{2EI^2}dV = \int_0^L \frac{M^2}{2EI^2}\left(y^2 dA\right)dx$$

$$U = \frac{M^2L}{2EI}$$

By energy conservation, the energy imparted to the spring by the axial loading is equal to that caused by the torsion and bending, and this equality forms the first line of Equations 5. Further substitution from Equations 4 completes the derivation for spring deflection due to an axial load. Using $J = \pi r^4 / 2$, $I = \pi r^4 / 4$ and $L = 2\pi n R \sec\alpha$, where n is the number of coils gives the useful equation for deflection, Equations 6. Note that when the spring is closely coiled (α is small), the simplified equation shown as Equation 7 is accurate.

$$U_{axial} = U_{torsion} + U_{bending}$$
$$\tfrac{1}{2}F\delta = \frac{T^2 L}{2GJ} + \frac{M^2 L}{2EI}$$
$$\tfrac{1}{2}F\delta = \frac{(FR\cos\alpha)^2 L}{2GJ} + \frac{(FR\sin\alpha)^2 L}{2EI}$$
$$\tfrac{1}{2}F\delta = \frac{F^2 R^2 L}{2}\left(\frac{\cos^2\alpha}{GJ} + \frac{\sin^2\alpha}{EI}\right)$$
$$\delta = FR^2 L\left(\frac{\cos^2\alpha}{GJ} + \frac{\sin^2\alpha}{EI}\right)$$

Equations 5. Energy balance and deflection equation of a loaded spring.

$$\delta = \frac{4FR^3 n\sec\alpha}{r^4}\left(\frac{\cos^2\alpha}{G} + \frac{2\sin^2\alpha}{E}\right)$$

Equation 6. Deflection of a loaded spring after substitution of moments of inertia.

$$\delta = \frac{4FR^3 n}{Gr^4}$$

Equation 7. Deflection of a loaded spring with

If Equation 7 for deflection and Equation 3 for shearing stress are compared, the shearing stress can be expressed as a function of the displacement, the modulus of rigidity, and the spring and coil dimensions, as shown in Equations 8. This equation is useful as one can check whether a certain spring can be allowed to undergo a certain deflection without failure. Comparing Equations 7 and Equation 8 now, an expression for δ can be cast in terms of τ. This is shown as Equation 9.

$$\tau = K\frac{2FR}{\pi r^3}$$
$$\delta = \frac{4FR^3 n}{Gr^4}$$
$$\frac{\tau}{\delta} = \frac{2FRK}{\pi r^3}\frac{Gr^4}{4FR^3 n}$$
$$\tau = K\delta\frac{Gr}{2\pi R^2 n}$$

Equations 8. Shear stress based on deflection.

Also of use in the automotive industry is the stiffness factor, S, for a given spring. This is simply the amount of force needed to create a unit of

$$S = \frac{F}{\delta} = \frac{F}{4FR^3 n / Gr^4} = \frac{Gr^4}{4R^3 n}$$

Equation 10. Spring stiffness.

$$\delta = \frac{4FR^3 n}{Gr^4}$$
$$\tau = \frac{1}{K}\cdot\frac{2FR}{\pi r^3}$$
$$\therefore \delta = \frac{1}{K}\cdot\frac{2n\pi\tau R^2}{Gr}$$

Equations 9. Deflection based on shear stress.

deflection and thus has the dimensions of force over length. Equation 10 shows the derivation of the stiffness factor.

Impact Loading

Impact loading occurs when a spring is given a sudden loading. What results is an incremental stress, $\Delta\tau$. Typically, this loading is characterized by the velocity of the incoming load, v. In order to find express $\Delta\tau$ in terms of v, Wahl first defines a surge wave velocity, v_S; its equation is defined as shown in Equation 11. This is the speed at which a wave is propagated through the spring. The propagation takes a finite time, Δt, to travel through the spring, causing an incremental deflection, $\Delta\delta$, and both are shown in Equations 12. Setting $\Delta\delta$ equal to δ in Equations 9 and solving for $\Delta\tau$ gives the incremental stress due to impact loading (Equations 13). In summary, the incremental deflection and stress $\Delta\delta$ (equations 12) and $\Delta\tau$ (13) should be taken into consideration when a spring undergoes impact loading.

$$v_s = \frac{r}{R}\sqrt{\frac{G}{2\rho}}$$

Equation 11. Surge wave speed of a impact loaded spring.

$$\Delta t = \frac{2\pi Rn}{v_s}$$
$$\Delta\delta = v\Delta t = v\frac{2\pi Rn}{v_s}$$
$$\Delta\delta = v\frac{2\pi Rn}{\frac{r}{R}\sqrt{\frac{G}{2\rho}}} = v\frac{2\pi R^2 n}{r}\sqrt{\frac{2\rho}{G}}$$

Equations 12. Time for surge wave to propagate through spring and the resultant incremental deflection.

$$\Delta\delta = v\frac{2\pi R^2 n}{r}\sqrt{\frac{2\rho}{G}}$$
$$\delta = \frac{1}{K}\cdot\frac{2n\pi\tau R^2}{Gr}$$
$$\Delta\delta = \delta$$
$$v\frac{2\pi R^2 n}{r}\sqrt{\frac{2\rho}{G}} = \frac{1}{K}\cdot\frac{2n\pi\tau R^2}{Gr}$$
$$\tau = \Delta\tau$$
$$Kv\sqrt{2\rho G}$$

Equations 13. Incremental stress due to impact loading.

Applying Theory to the Automobile Suspension System

Having derived equations for the maximum shearing stress of a spring, the deflection under load and spring stiffness, the automotive suspension can be considered in more detail. Because the tires and chassis contribute also to the overall deflection of the wheels, formally, these should also be taken in consideration. However, Gillespie points out that the spring rate of a tire is many times greater than that of the typical automotive suspension helical spring, and because the tire and spring are in series, using the stiffness of the spring alone can approximate the spring rate of the vehicle. Another error in the simplification of the suspension system is that, in most cases, the load caused by a bump is not directly inline with the spring's coil axis. Consequently, the axial load, F, seen by the spring is often not the load that the bump causes directly; other suspension members such as the control arm act as a lever arm.

Statically Loaded Front Axle

As mentioned in the introduction, I have designed an example of a statically loaded front axle. Throughout the examples, an effort has been made to keep all numbers (geometry, size, etc.) representative of those found on a passenger automobile.

> A suspension designer wishes to reduce unsprung weight of a vehicle by replacing a vehicle's front steel helical springs with aluminum alloy springs without changing the spring stiffness, the coil diameter (D = 14 cm) or the number of coils per spring. The vehicle weighs 1500 kg, with 60% of its weight resting over its front wheels statically. Assume that the helix angle is 8° or less such that Equation 2 can be used with adequate accuracy. Help determine if this change should be allowed and if it is beneficial by performing the following calculations:
>
> a) Determine the weight over each front wheel that has to be supported by the springs.

$$\text{Weight}_{\text{front wheel}} = 1500kg \cdot \frac{9.81N}{kg} \cdot \frac{0.60}{axle} \cdot \frac{axle}{2wheels} = \frac{4415N}{wheel}$$

b) Determine the spring element radius for the current design if the spring stiffness is $S = 20000\, {}^{N}\!/\!{}_{m}$. Use $G_{steel} = 77GPa$.

$$S = \frac{Gr^4}{4R^3 n}$$

$$r = \left(\frac{4SR^3 n}{G}\right)^{\frac{1}{4}} = \left(\frac{4 \cdot 20000\, {}^{N}\!/\!{}_{m} \cdot (0.07m)^3 \cdot 8}{77\times10^9\, Pa}\right)^{\frac{1}{4}} = 7.31mm$$

c) Using Equations 3 for the correction factor, calculate the maximum shearing stress of the steel spring if the front axle lies over a bump 8 cm high and that the car raises by 4 cm when resting over the bump (such that the deflection from a car at rest is 4 cm). Also determine the factor of safety for the shearing stress, assuming that the ultimate stress in shear for the steel is 900 MPa.

$$K = \frac{4\,{}^{R}\!/\!{}_{r} - 1}{4\,{}^{R}\!/\!{}_{r} - 4} + \frac{0.615}{{}^{R}\!/\!{}_{r}} = \frac{4\,{}^{0.07m}\!/\!{}_{7.31\times10^{-3}m} - 1}{4\,{}^{0.07m}\!/\!{}_{7.31\times10^{-3}m} - 4} + \frac{0.615}{{}^{0.07m}\!/\!{}_{7.31\times10^{-3}m}} = 1.15$$

$$\tau_{bump} = K\frac{2FR}{\pi r^3} = K\frac{2(S\delta)R}{\pi r^3} = 1.15\frac{2\left(20000\,{}^{N}\!/\!{}_{m}(0.08m - 0.04m)\right)\cdot 0.07m}{\pi\left(7.31\times10^{-3}m\right)^3} = 105.0MPa$$

$$\tau_{static} = K\frac{2FR}{\pi r^3} = 1.15\frac{2\cdot 4415N \cdot 0.07m}{\pi\left(7.31\times10^{-3}m\right)^3} = 579.2MPa$$

$$\tau = \tau_{bump} + \tau_{static} = 105.0MPa + 579.2MPa = 684.2MPa$$

$$F.S = \frac{\tau_{ultimate}}{\tau} = \frac{900MPa}{684.2MPa} = 1.32$$

Comment: For the above representative automotive suspension spring, the correction factor is important as it raises the shearing stress by about 15%. τ_{bump} is the shearing stress due to the 4 cm of deflection caused by the bump, while τ_{static} is the shearing stress from the weight supported by a front wheel without the existence of the bump. The total shearing stress is the sum of both these values.

d) Find the mass of a steel spring by first calculating the spring volume and using $\rho_{steel} = 7860\, {}^{kg}\!/\!{}_{m^3}$.

$$m = \rho V = \rho AL = \rho\left(\pi r^2\right)\left(2\pi nR\sec\alpha\right)$$

$$m = 7860\,{}^{kg}\!/\!{}_{m^3}\left(\pi\left(7.31\times10^{-3}m\right)^2\right)\left(2\pi(8)\cdot 0.07m\cdot\sec(8°)\right)$$

$$m = 4.69kg$$

Comment: The 8° angle is arbitrary and will affect the mass calculated. To determine the angle exactly, more will need to be known about the geometry of the suspension system. Furthermore, the angle will vary with loading. The choice of 8° allows the derived formulas to be used with confidence of moderate accuracy.

e) For aluminum alloy, recalculate the spring element radius necessary to maintain the same stiffness, using $G_{Al} = 26GPa$.

$$S = \frac{Gr^4}{4R^3 n}$$

$$r = \left(\frac{4SR^3 n}{G}\right)^{\frac{1}{4}} = \left(\frac{4 \cdot 20000\, {}^{N}\!/_{m} \cdot (0.07m)^3 \cdot 8}{26\times 10^9\, Pa}\right)^{\frac{1}{4}} = 9.59mm$$

Comment: Due to the reduced modulus of rigidity for aluminum, a larger spring element is necessary to maintain the original stiffness of 20000 N/m.

f) Recalculate the correction factor, maximum shearing stress and the new factor of safety when aluminum alloy is used.

$$K = \frac{4\,{}^{R}\!/_{r} - 1}{4\,{}^{R}\!/_{r} - 4} + \frac{0.615}{{}^{R}\!/_{r}} = \frac{4\,{}^{0.07m}\!/_{9.59\times 10^{-3}m} - 1}{4\,{}^{0.07m}\!/_{9.59\times 10^{-3}m} - 4} + \frac{0.615}{{}^{0.07m}\!/_{9.59\times 10^{-3}m}} = 1.20$$

$$\tau_{bump} = K\frac{2FR}{\pi r^3} = K\frac{2(S\delta)R}{\pi r^3} = 1.20\frac{2\left(20000\,{}^{N}\!/_{m}\,(0.08m - 0.04m)\right)\cdot 0.07m}{\pi\left(9.59\times 10^{-3}m\right)^3} = 48.5MPa$$

$$\tau_{static} = K\frac{2FR}{\pi r^3} = 1.20\frac{2 \cdot 4415N \cdot 0.07m}{\pi\left(9.59m\times 10^{-3}m\right)^3} = 267.7MPa$$

$$\tau = \tau_{bump} + \tau_{static} = 48.5MPa + 267.7MPa = 316.2MPa$$

$$F.S = \frac{\tau_{ultimate}}{\tau} = \frac{240MPa}{316.2MPa} = 0.76$$

Comment: Due to the increased spring element radius, the correction factor has also increased. More importantly, the above calculations show that the *aluminum is unable to meet the original specifications of spring stiffness and geometry without failure*, and that aluminum is not a viable option. It would be necessary to modify the coil's geometry in order to both retain the stiffness and be of sufficient strength.

f) Estimate the percentage reduction in spring weight, if any, using $\rho_{Al} = 2770 \,{}^{kg}\!/\!_{m^3}$.

$$m_{Al} = \rho V = \rho AL = \rho(\pi r^2)(2\pi nR\sec\alpha)$$
$$m_{Al} = 2770 \,{}^{kg}\!/\!_{m^3}\left(\pi(9.59\times10^{-3}m)^2\right)(2\pi(8)\cdot 0.07m\cdot\sec(8°))$$
$$m_{Al} = 2.84kg$$
$$\%_{reduction} = \left(1-\frac{m_{steel}-m_{Al}}{m_{steel}}\right)\cdot 100\% = 60.6\%$$

Comment: The above shows that if the aluminum were able to support the load, there would be a significant weight savings.

Bump Impact at Significant Speed

When the load is not static but dynamic, such as when the bump is hit while the vehicle is in motion, there exists an incremental shearing stress due to the impact loading. The following example will show the effects of impact loading.

Consider the previous example. Now, the designer wishes to determine the maximum speed at which a similar bump (one that raises the axle 4 cm) can be approached without spring failure by shear. Use the same properties for steel.

a) Determine how much incremental impact shearing stress, $\Delta\tau$, can be applied if a minimum factor of safety of 1.1 is prescribed.

$$\tau_{impact} = \frac{\tau_{ultimate}}{F.S}$$
$$\tau + \Delta\tau = \frac{\tau_{ultimate}}{F.S}$$
$$\Delta\tau = \frac{\tau_{ultimate}}{F.S} - \tau$$
$$\Delta\tau = \frac{900MPa}{1.1} - 684.2MPa = 134.0MPa$$

Comment: The incremental stress due to impact loading limits the speed at which the car can be driven over the bump.

b) Using Equations 12, determine v_{spring}, the maximum average speed at which the spring can be allowed to deflect for the $\Delta\tau$ calculated earlier.

$$\Delta\tau = Kv_{spring}\sqrt{2\rho G}$$
$$v_{spring} = \frac{\Delta\tau}{K\sqrt{2\rho G}} = \frac{134.0\times10^{6}\,Pa}{1.15\sqrt{2\cdot 7860\,{}^{kg}\!/\!_{m^3}\cdot 77\times10^{9}\,Pa}} = 3.35\,{}^{m}\!/\!_{s}$$

Comment: In this part of the example, a rearrangement of Equations 12 is used to determine the maximum speed at which the spring can be deflected in order to meet the factor of safety requirement.

c) Using Figure 5, determine the maximum speed at which the car can be driven. Take the radius of the wheel/tire combination, r_{wheel} to be 0.3 m and the effective height of the bump (the necessary deflection), h, to be 0.04 m. Take the width of the bump to be 0.10 m.

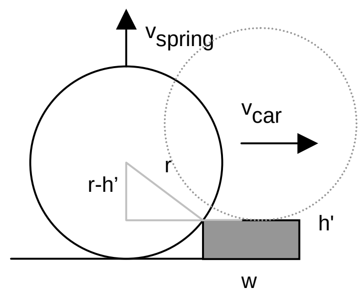

Figure 5. Schematic of wheel traveling over bump at speed.

$$v_{spring} = \frac{h}{t} = \frac{h}{d/v_{car}} = \frac{h\cdot v_{car}}{d}$$
$$v_{car} = \frac{v_{spring}\cdot d}{h} = \frac{v_{spring}\left(\left(r_{wheel}^{\;2}-(r_{wheel}-h)^2\right)^{\frac{1}{2}}+\frac{w}{2}\right)}{0.04m} = \frac{3.35\,{}^{m}\!/\!_{s}\left(\left((0.3m)^2-(0.3m-0.04m)^2\right)^{\frac{1}{2}}+\frac{0.1m}{2}\right)}{0.04m}$$
$$v_{car} = 16.7\,{}^{m}\!/\!_{s} = 60.2\,{}^{km}\!/\!_{h} = 37.4mph$$

Comment: Several assumptions were made in this step. First, the calculated value of v_{spring}, the maximum average speed of spring deflection was used to determine the speed of the car. It is likely that the absolute maximum spring deflection speed is significantly higher, overestimating the car's highest permissible speed. The above formula is derived by equating the time needed for the 0.04 m deflection to the time needed for the car and wheel to travel half the width of the bump. Hence, the width of the bump also affects the solution.

One assumption that may not be entirely appropriate is the neglect of the shock absorbers. Most shock absorbers have gases at a low enough pressure so that they do not provide any supporting or springing effect when the car's wheels are not deflected. However, in this case, the wheel is deflected quickly, and the shocks will have a stiffening effect, increasing the overall spring rate. Nevertheless, this example shows that impact loading needs to be considered in spring design.

Conclusions

This project has covered several aspects of the helical spring in the context of an automobile's suspension. First, internal forces and torsion were used to derive the shearing stress of a loaded spring. It was noted that the shearing stress is a result of a direct shear and torsion, and that it is common to use only the torsional stress component explicitly, while a correction factor is used to take care of the direct shear. An energy method analysis followed, and using the internal energy due to axial loading, torsion and bending, an expression for deflection was written in terms of the axial load. This was simplified assuming that the helix inclination angle was small. Manipulations and comparisons of these equations allowed the determination of the shearing stress and deflection in terms of other variables as well as a formula for spring stiffness. Assuming that a surge wave propagated through the spring, expressions for incremental deflection and incremental shear stress were derived for the case of impact loading where the velocity of the impact with respect to the spring is known.

Following the theoretical analysis were two examples, showing static loading and impact loading. In the first example, a switch to aluminum meant that it was impossible to keep the coil dimensions unchanged if the spring stiffness were to be held constant even though a significant weight saving could come from the use of aluminum. The second example showed that impact loading limited the maximum speed at which a car can be driven over a bump due to the incremental stress and deflection.

Finally, it should be noted that this project covers only the basics of the helical spring. In most calculations, several assumptions were made. They included that the helix angle be small and that the shear stresses be distributed evenly over the spring element's cross-section (in essence neglecting curvature). Fatiguing, lateral loading and temperature effects were entirely ignored. Furthermore, other dynamic effects such as resonance and spring clashing were not discussed in this report, and so neither was the fact that the spring is not the only energy absorbing device in the automobile's suspension system.

Despite ignoring the above factors, the level of analysis appeared to generate answers in the correct order of magnitude. However, it appears that the internal shearing stresses calculated were rather high. Perhaps the analysis led to an overestimation of the internal shearing stresses. Also likely is that the material property data were not fully representative of those used as automotive suspension springs. The real materials used are likely laboriously treated to allow for large deflections without significant creeping, fatiguing or failure. Nevertheless, the theory covered and the calculations performed appear to have been sufficient for a preliminary analysis of the helical spring in a car suspension.

References

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